ROD CUTTING

Given a rod of length n inches and an array of prices that contains prices of all pieces of size smaller than n. Determine the maximum value obtainable by cutting up the rod and selling the pieces.
For example, if length of the rod is 8 and the values of different pieces are given as following, then the maximum obtainable value is 22 (by cutting in two pieces of lengths 2 and 6)

length	0	1	2 ✓	3	4	5	6 ✓	7	8
price	0	1	5	8	9	10	17	17	20

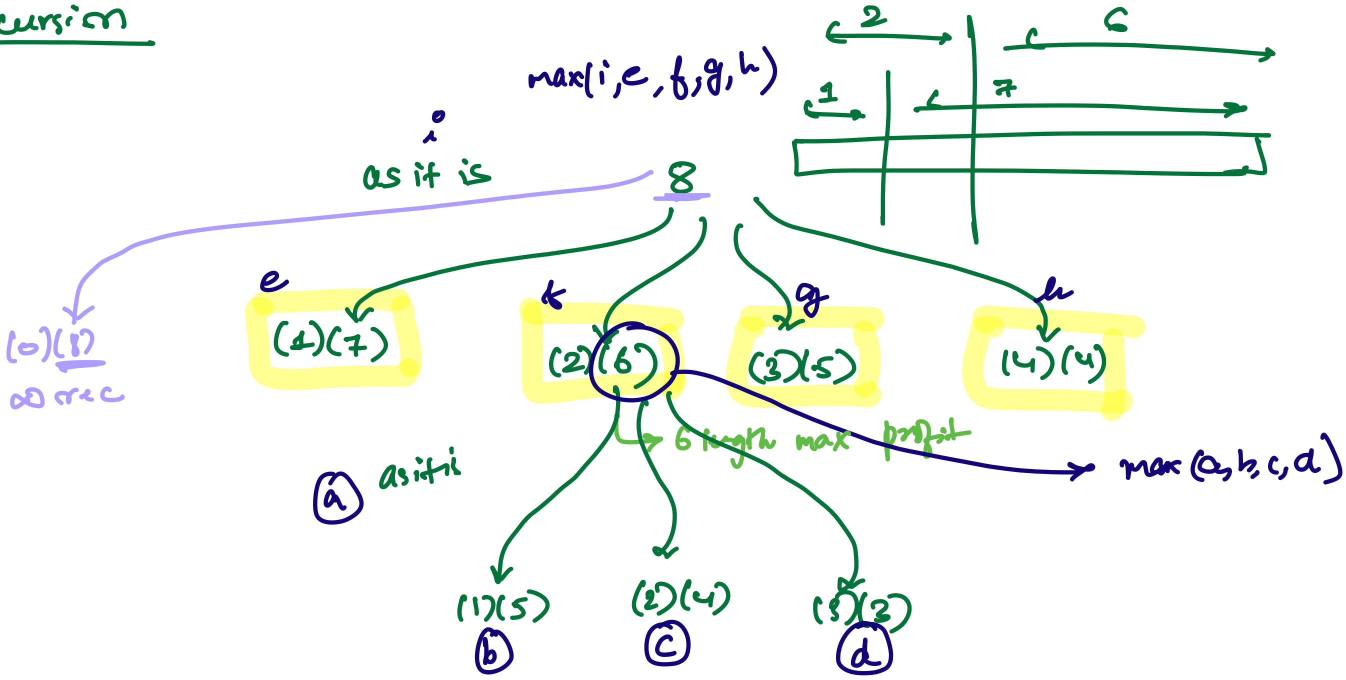
5+17 = 22 4+4 = 18

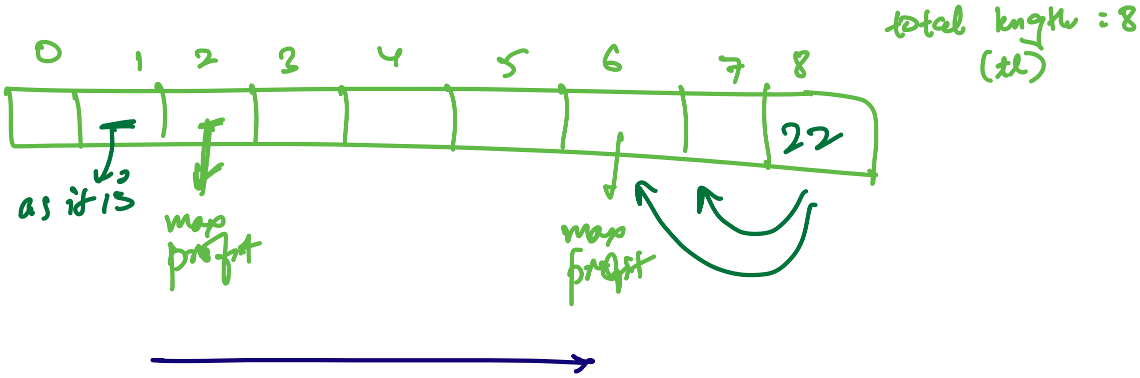
And if the prices are as following, then the maximum obtainable value is 24 (by cutting in eight pieces of length 1)

length	0	1	2	3	4	5	6	7	8
price	0	3	5	8	9	10	17	17	20

3x8=24

Recursion





BU:

- storing ? n=8
  n+1 size

- TD BC return
  BU fill

- cell meaning

- filling dir

CATALAN NUMBER

- first few catalan numbers are:

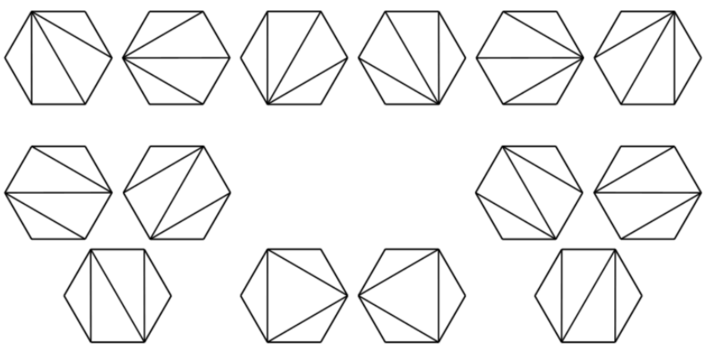
n	0	1	2	3	4	5	6	7	8	9	....
Catalan No.	1,	1,	2,	5,	14,	42,	132,	429,	1430,	4862	.....

Applications:

1. Number of possible Binary Search Trees with n keys.

2. Number of expressions containing n pairs of parentheses which are correctly matched. For n = 3, possible expressions are ((())), ()(()), ()()(), (())(), (()()).

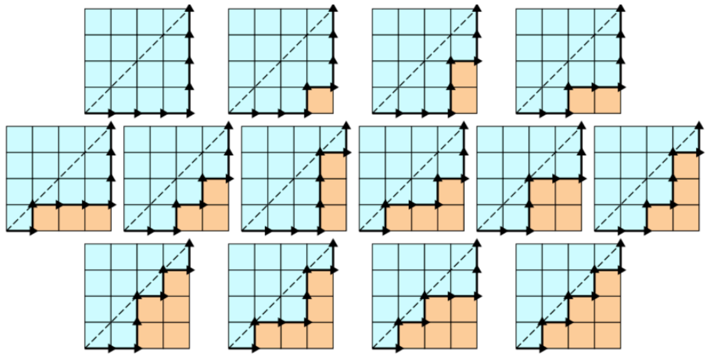
3. Number of ways a convex polygon of n+2 sides can split into triangles by connecting vertices.



4. Number of full binary trees (A rooted binary tree is full if every vertex has either two children or no children) with n+1 leaves.

5. Number of different Unlabelled Binary Trees can be there with n nodes.

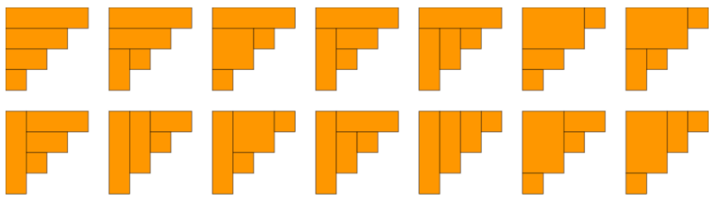
6. The number of paths with 2n steps on a rectangular grid from bottom left, i.e., (n-1, 0) to top right (0, n-1) that do not cross above the main diagonal.



7. Number of ways to insert n pairs of parentheses in a word of n+1 letters, e.g., for n=2 there are 2 ways: ((ab)c) or (a(bc)).
For n=3 there are 5 ways, ((ab)(cd)), (((ab)c)d), ((a(bc))d), (a((bc)d)), (a(b(cd))).

8. Number of Dyck words of length 2n. A Dyck word is a string consisting of n X's and n Y's such that no initial segment of the string has more Y's than X's. For example, the following are the Dyck words of length 6: XXXYYY XYXXYY XYXYXY XXYYXY XXYXYY.

9. Number of ways to tile a stairstep shape of height n with n rectangles. The following figure illustrates the case n = 4:



10. Number of ways to form a "mountain ranges" with n upstrokes and n down-strokes that all stay above the original line.The mountain range interpretation is that the mountains will never go below the horizon.

Mountain Ranges

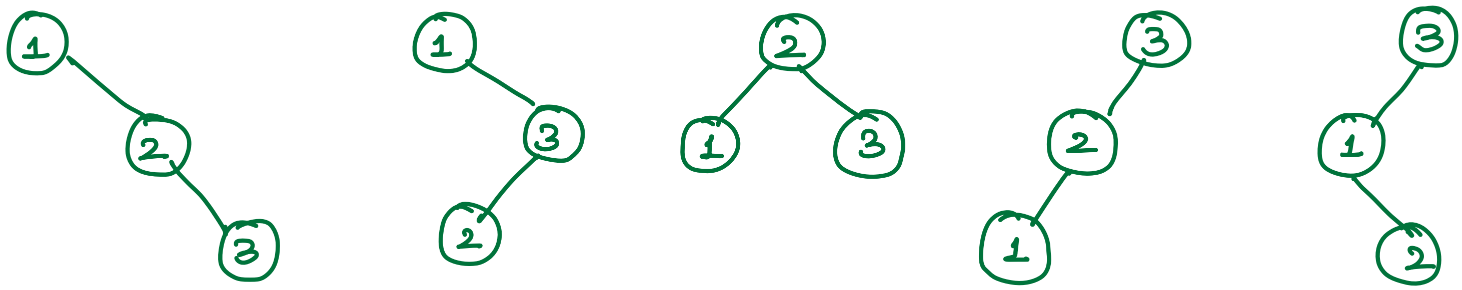
$n = 0:$	*	1 way
$n = 1:$	/\	1 way
$n = 2:$	/\ /\/\, / \	2 ways
$n = 3:$	/\ /\ /\ /\/\ / \ /\/\/\, /\/ \, / \/\, / \, / \	5 ways

No. of unique BSTs:

* If $n=3$

nodes : 1, 2, 3

no of bsts = 5



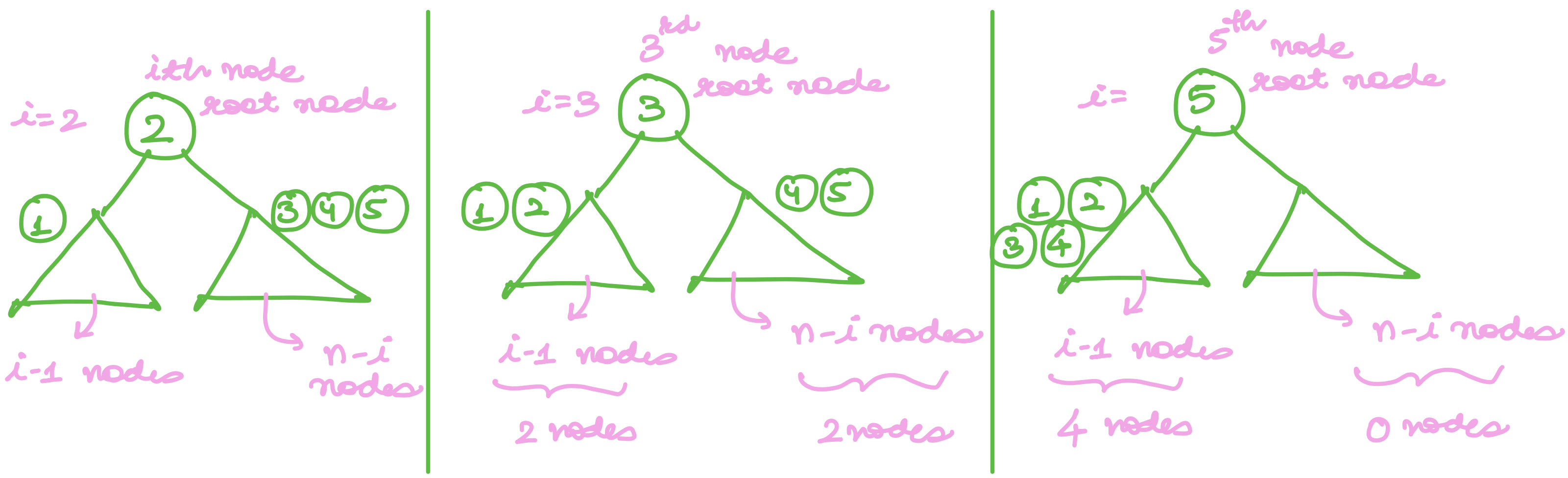
* If $n=5$

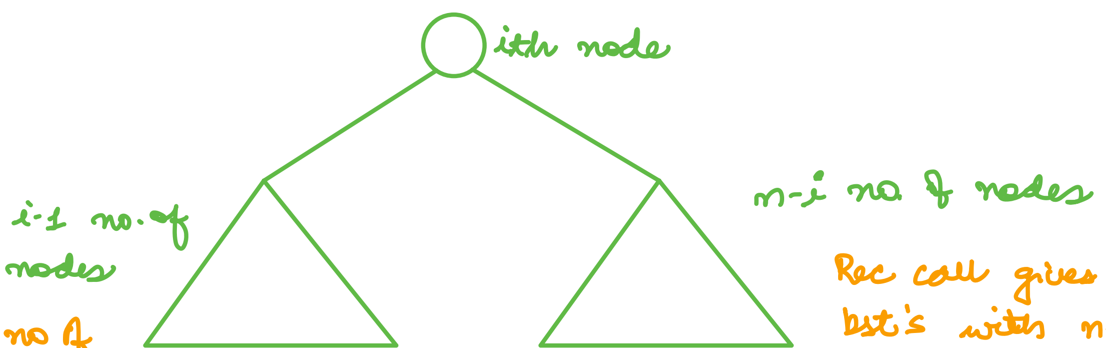
nodes: 1, 2, 3, 4, 5

$$\underbrace{1 \text{ root node}}_{a \text{ bsts}} + \underbrace{2 \text{ root node}}_{b \text{ bsts}} + \underbrace{3 \text{ root node}}_{c \text{ bsts}} + \underbrace{4 \text{ root node}}_{d \text{ bsts}} + \underbrace{5 \text{ root node}}_{e \text{ bsts}}$$

Total bst's = $a+b+c+d+e$

$$\sum_{i=1}^{n} \text{ith root node bsts}$$

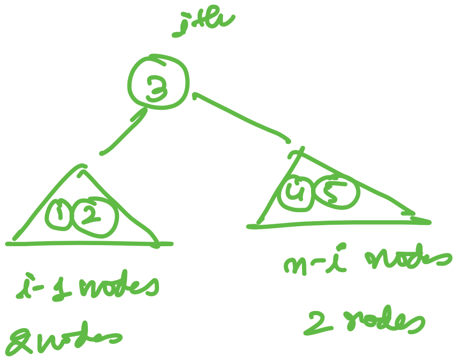




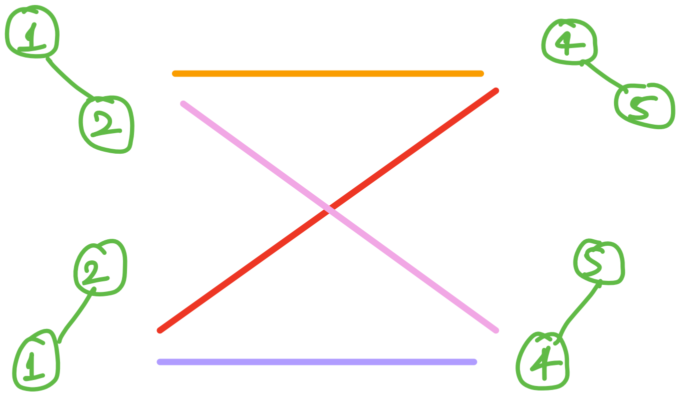
Rec call gives no of bsts with i-1 nodes

Rec call gives no. of bst's with n-i nodes.

1,2,3,4,5



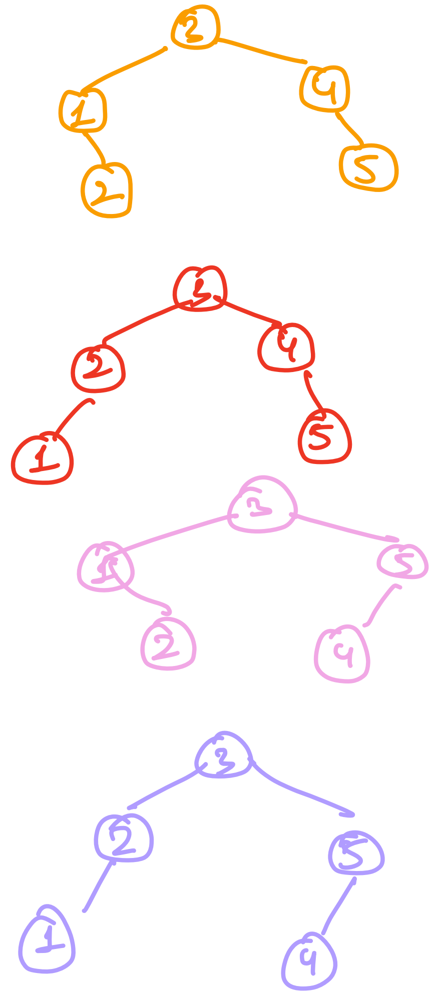
1 2 arrangement: 4 5



x bst's → output of Rec call ← y bst's

Rec call: input: no of nodes : 2 nodes
output: no of bsts : 2 bsts

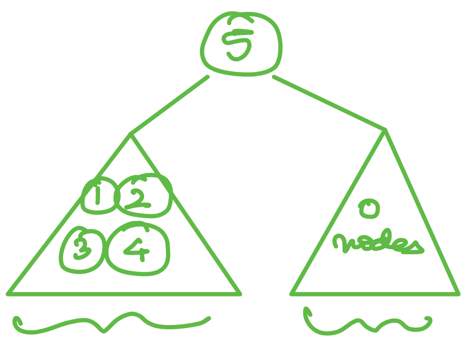
Total bst's = x*y



x*y

Base case:

i=5



x bst's * 1 = x bst's

if we return 0 from b.c (i.e. RC(0)=0) then x bst's * 0 = 0